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Self-field and wake of a charged particle in a plasma

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The self-field and the associated plasma response of an externally injected charged particle in plasmas are investigated within a self-consistent formalism that extends a formula used in plasma wake-field acceleration. The response of plasmas to such a self-field is also investigated through this self-consistent theory. This self-consistent theory leads to a correct formula of plasma wake field.

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It is well known that, for a charged point particle in vacuum, its electric field includes two terms: self-field and radiation field [1]. If the particle is in uniform motion, its electric field is identical to the self-field term, which moves with same velocity as particle velocity βc. Because charged particles in uniform motion possess self-fields moving at a fixed velocity, they are often used to excite plasma wave of the expected phase velocity [2–6]. Here, the expected phase velocity is equal to the moving velocity of the driving field. When studying the excitation of plasma wave by a bunch of charged particles at a fixed velocity βc, we have to deal with the self-field of a driver particle in plasmas. This issue is crucial for understanding correctly the physics of excited plasma wake field.

In this work, we develop a self-consistent theory for the self-field of an externally injected particle in plasmas. In many published works [2,3], plasma response to this self-field, or plasma wake field, is described elegantly by a fluid equation. Despite this equation captures the main physics behind a complex phenomenon, however, the correct or reasonable solution of this equation is not yet reached. If a correct or reasonable solution is presented in Ref. [2], a universal theory for plasma wake field at any phase velocity would have been established in the mid of 1980s. Unfortunately, the solution presented in Ref. [2] seems unsuccessful because it is not consistent with the basic assumption on this equation. This unsuccessful solution greatly limits the theory in Ref. [2] to be only valid at the $\beta \sim 1$ case. This will lead to many incorrect viewpoints on plasma wake field. The purpose of this work is to obtain a correct form of plasma wake field from a self-consistent theory on the self-field of an externally injected particle in plasmas.

For convenience, we define the self-field of a charged particle in vacuum as bare self-field and denote it as E^0_{self}. E^0_{self} fulfills the following formulas:

$$E^0_{self}|_{\xi\neq 0} \propto \frac{\xi}{|\xi|}\frac{1}{\xi^2}, \quad E^0_{self}|_{\xi=0} = 0,$$

$$[\nabla \cdot E^0_{self}]|_{\xi\neq 0} = 0, \quad [\nabla \cdot E^0_{self}]|_{\xi=0} = \delta(\xi) \neq 0, \tag{1}$$

where $\xi = z - \beta ct$, βc is the particle's velocity, and $\xi=0$ denotes the axial position of the point charge. In contrast, its self-field in plasmas meets the similar equation

$$[\nabla \cdot E_{self}]|_{\xi\neq 0} = 0, \quad [\nabla \cdot E_{self}]|_{\xi=0} = \delta(\xi) \neq 0. \tag{2}$$

However, except for E_{self}, the total self-field also includes the contribution from plasma electrons E_{pl}, which meets

$$[\nabla \cdot E_{pl}] = n_{pl}(\xi), \tag{3}$$

where $n_{pl}(\xi)$ stands for plasma charge density generated from the interaction. For unperturbed plasmas, there is $n_{pl}(\xi,\tau) = 0$. For any position ξ, whether $n_{pl}(\xi,\tau)$ can derivate from 0 is determined by the total self-field $[E_{self}+E_{pl}](\xi,\tau)$ at this point. The force balance condition, $[E_{self}+E_{pl}](\xi,\tau)=0$, could ensure $n_{pl}(\xi,\tau)=0$ at any time τ. Therefore, two conditions

$$[E_{self} + E_{pl}](\xi,\tau) = 0, \quad \nabla \cdot [E_{self} + E_{pl}](\xi,\tau) = 0 \tag{4}$$

define the unperturbed plasma region.

Note that the equation $[\nabla \cdot f] = \delta(\xi)$ could have multiple solutions. Except for the well-known solution $f|_{\xi\neq 0} \propto (\xi/|\xi|)(1/\xi^2)$ and $f|_{\xi=0}=0$, other solutions, such as $f(\xi \neq 0) = f(\xi=0^+)$, are also possible. Both E_{self} and E^0_{self} satisfy the same differential equation $[\nabla \cdot f] = \delta(\xi)$, and they are associated with different conditions for determining the solution. If the unperturbed region starts from $\xi=0$ and the solution of $E_{self}+E_{pl}$ in the unperturbed region is described by $f(\xi\neq 0) = f(\xi=0^+)$, the above two conditions will require $E_{pl}(\xi\neq 0) = -E_{self}(\xi \neq 0) = -E_{self}(\xi=0^+)$. Because the self-field of any point charge on itself should be zero, there is $E_{self}(\xi=0)=0$. For a point charge, there should be a jump from $E_{self}(\xi=0)$ to $E_{self}(\xi=0^+)$ because of $[\nabla \cdot E_{self}]|_{\xi=0} = \delta(\xi)$. The above two conditions also require that E_{pl} has a similar jump (but of opposite sign) and n_{pl} takes a Dirac-like shape at $\xi=0$. Therefore, this implies that if the unperturbed region starts from $\xi=0$, there must be $n_{pl}(\xi=0)\neq 0$. Thus, when ξ transits from 0 to 0^+, E_{pl} undergoes a jump similar to that of E_{self} (but of opposite sign) and hence $[E_{self}+E_{pl}](\xi,\tau)=0$ is valid. Moreover, a static external particle of charge Q expels plasma particles whose charge amount is equal to Q, and hence causes n_{pl} at $\xi=0$ exhibiting a Dirac-function-type valley (but the total charge density, $\delta(\xi) + n_{pl}(\xi)$, is still zero at $\xi=0$). Those expelled plasma charges distribute over the whole space and hence correspond to a negligible plasma charge density in the $\xi \neq 0$ region (i.e., $\frac{Q}{\text{volume}\to\infty} \sim 0$).

When we investigate the interaction of a charged particle with plasmas, it is not necessary to know exactly E_{self} because we are interested in n_{pl}. Except for $\xi=0$, there is $n_{pl} = \nabla \cdot [E_{self}+E_{pl}]$. A differential equation of n_{pl} in the perturbed region could be found in Ref. [2],

$$\partial_{\xi\xi} n_{pl} + k_p^2 n_{pl} = k_p^2 \delta(\xi)/e, \tag{5}$$

where $k_p \equiv \omega_p/\beta c$ and ω_p is plasma circular frequency. According to Ref. [2], this equation is based on an assumption that *the change in velocity of the driving particle* is "negligible," i.e., $d_t\beta=0$. This assumption implicitly requires $[E_{self}+E_{pl}](\xi=0)=0$. Moreover, how to determine the condition for determining solution of this differential equation is a worthy question. When considering the condition for determining solution, we should take into account the above two conditions for the unperturbed region. A solution of Eq. (5) is presented in Ref. [2] as

$$n_{pl}(\xi<0) = \frac{k_p}{e}\sin(k_p\xi), \quad n_{pl}(\xi>0)=0. \tag{6}$$

The condition for determining this solution is $[E_{self}+E_{pl}](\xi=0)=\cos(0)\neq 0$ and the dichotomy between the unperturbed and the perturbed regions is located at $\xi=0$. Obviously, $[E_{self}+E_{pl}](\xi=0)=\cos(0)\neq 0$ is not consistent with the above-mentioned assumption. Moreover, this dichotomy corresponds to $\nabla \cdot [E_{self}+E_{pl}](\xi=0)=\delta(\xi)\neq 0$ and hence contradicts the above two conditions for the unperturbed region.

The correct form of the condition for determining solution should be

$$[E_{self}+E_{pl}](\xi = z_{boundary}, \tau) = 0,$$

$$\nabla \cdot [E_{self}+E_{pl}](\xi = z_{boundary}, \tau) = 0,$$

$$[E_{self}+E_{pl}](\xi=0) = 0, \tag{7}$$

where $z_{boundary}$ stands for the position of the dichotomy, $\xi > z_{boundary}$ is for unperturbed region, and $\xi < z_{boundary}$ is for perturbed region. A solution of Eq. (5)

$$n_{pl}\left(\xi < \frac{\pi}{2k_p}\right) = \frac{k_p}{e}\sin\left[k_p\left(\xi - \frac{\pi}{2k_p}\right)\right] = -\frac{k_p}{e}\cos(k_p\xi), \tag{8}$$

$$n_{pl}\left(\xi > \frac{\pi}{2k_p}\right) = 0$$

could satisfy these conditions and corresponds to $z_{boundary} = \pi/(2k_p) = [\beta c/(2\omega_p)]\pi$. Indeed, because of $\partial_\xi = \partial_{\xi'}$ for $\xi' = \xi + l$, Eq. (5) could be expressed as $\partial_{\xi'\xi'} n_{pl}(\xi') + k_p^2 n_{pl}(\xi') = k_p^2\delta(\xi'-l)/e$. This also implies that the solution of Eq. (5) is not unique. Solution (8) could be obtained by shifting solution (6) along the ξ axis with a distance $\pi/(2k_p)$.

Since two solutions are very much alike in perturbed plasmas, it seems trivial to exactly determine which one of two solutions describes the plasma wake of a charged particle. Unfortunately, this is not a trivial task for wake-field acceleration and excitation. If the wake field of a charged particle is described by solution (6), energy balance requires that the increment in the wake-field energy $E_{pl}^2 \upsilon t A$ is equal to the decrement in the kinetic energy of driving particle $\gamma_q m_e c^2$, i.e., $d_t E_{pl}^2 \upsilon t A = -d_t \gamma_q m_e c^2$ [2]. Therefore, solution (6) determines that the excitation of wake field requires inevitably the deceleration of the driving particle. As pointed out above, this contradicts the assumption $d_t\beta=0$ [2].

Because of $\nabla \cdot E_{pl} = -en_{pl}$, $E_{pl}^2 \upsilon t A$ is indeed a portion of total potential energy $\int[-en_{pl}+q\delta(\xi)][\phi_{pl}+\phi_{self}]d\xi$, where q is the charge of the driver particle. Strictly speaking, the energy balance condition should be written as

$$\begin{aligned} 0 &= d_t[\text{total kinetic energy} + \text{total potential energy}] \\ &= d_t\Big[E_{pl}^2 \upsilon t A + q\phi_{pl}(0,\tau) + \int -en_{pl}\phi_{self}d\xi + q\phi_{self}(0,\tau) \\ &\quad + \gamma_q m_e c^2 + \int n_{pl}\gamma(\upsilon_{pl}) m_e c^2 d\xi\Big]. \end{aligned} \tag{9}$$

Since solution (6) corresponds to $\phi_{pl}(0,\tau)=0$, this implies that the increment in E_{pl}^2 is inevitably accompanied by the decrement in $\gamma_q m_e c^2$. In contrast, if the wake field is described by solution (8), we will find that the energy balance condition could be valid even when $\gamma_q m_e c^2$ is conserved. In such a situation, $d_t[E_{pl}^2 \upsilon t A]>0$ could be valid if $d_t q\phi_{pl}(0) < 0$ exists. For solution (8), the deformation effect in perturbed plasmas is included and hence ϕ_{pl} is time dependent in the β frame, only the boundary condition restricting $\phi_{pl}(\xi=\pi/(2k_p),\tau)=0$.

We have developed a self-consistent theory on the self-field of an externally injected particle in plasmas. This self-consistent theory indicates that the dichotomy between unperturbed and perturbed plasmas does not automatically locate at the particle's position but at a position ahead of the particle. The position of the dichotomy depends on the particle's velocity. Because plasma wake field is the response of plasmas to the self-field of the driver particle, rather than to the driver particle itself, the causality does not require that the dichotomy must be at $\xi=0$. Strict analysis indicates that the causality requires the dichotomy to be located at $z_{boundary} < \infty$. Because the dichotomy is correctly determined, this self-consistent theory will lead to a correct formula of the plasma wake field of a bunch of charged particles at a fixed finite velocity.

The significance of our theory is that it extends a well-known fluid theory for plasma wake field [2] with universal validity. More important, our theory enables the fluid theory for plasma wake field to be consistent with the result of various advanced self-consistent simulations. For instance, when we apply particle-in-cell (PIC) simulation to study the excitation of plasma wake field, it is difficult to find the consistency between simulation results and fluid theory presented in Ref. [2]. This is because in PIC simulation every macroparticle contributes to the self-consistent field at the position in front of itself, whereas in the fluid theory [2] every particle does not contribute to the self-consistent field in front of itself. Moreover, the above-mentioned correct energy balance relation reveals that the increment in the strength of a plasma wake field could be achieved through the deceleration of the driver particles. This is indeed a theoretical confirmation of a viewpoint that a driver particle could excite a plasma wake field whose phase velocity is equal to the particle's velocity. According to Ref. [2], the increment in the strength of a plasma wake field must be realized through the deceleration of the driver particles, and hence this indeed implies that a driver particle cannot excite a plasma wake field of fixed phase velocity. In short, we have reached a sound improvement on the fluid theory in Ref. [2]. Such an improved fluid theory is of practical value to interpret correctly the associated experimental and simulation results.

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